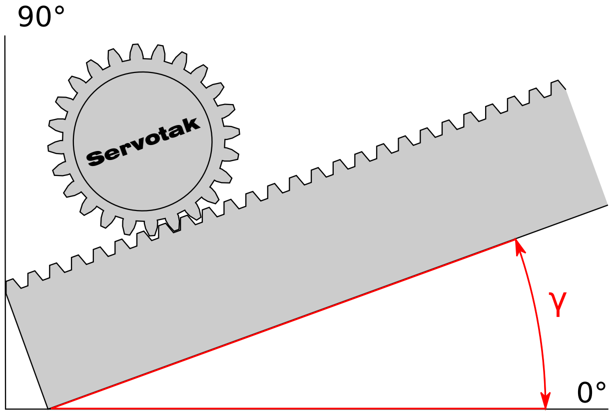
Rack & Pinion System



Rack & Pinion System

The rack and pinion mechanism converts rotational motion into linear motion. It offers both high precision and high cyclic rates. They're used for Cartesian coordinate robots, robotics, gantry type machining centers, movable column milling machines, lathes, simulators and test benches, elevators, machine tools, etc. Servotak precision rack and pinion offer multiple advantages compared to other manufacturers.

Disclaimer

This tool has been created to assist engineers with the sizing of the different parts of the system. Calculations might not cover all corner cases. and results should always be checked by a qualified engineer. Under no circumstances shall we beheld responsible to any damages to persons or property due to correct or incorrect use of this tool, or to errors in it.

Cycle Duration

$$t_{cycle} = t_a + t_{cs} + t_d + t_{dw} \quad [s]$$

Cycles per Minute

$$Z = \frac{60}{t_{cycle}}$$

Duty Cycle

$$Z = \frac{t_a + t_{cs} + t_d}{t_{cycle}}$$

Friction Force

$$F_f = \frac{m \cdot \mu \cdot g \cdot \cos\left(\frac{\gamma \cdot \pi}{180}\right)}{\eta} \quad [N]$$

Acceleration Force

$$F_{acc} = \frac{m \cdot a}{\eta} \quad [N]$$

Constants

Pi	$\pi \simeq 3.141592654$
Acceleration of Gravity on Earth	$g = 9.80665 \; \frac{m}{s^2}$

Inputs

Acceleration Time	$t_a \; [s]$
Deceleration Time	$t_d \; [s]$
Constant Speed Time	$t_{cs} \; [s]$
Dwell Time	$t_{dw} \; [s]$
System Inclination	$\gamma \; [degrees]$
Traveling Weight	$m \; [kg]$
Speed	$v \; \left[\frac{m}{s}\right]$
Static Friction Coefficient	μ_s
Service Factor	K_A
System Efficiency	η
Pinion Pitch Diameter	$d \; [mm]$
Pinion Inertia	$J_P \; [kg \cdot cm^2]$
Pinion Pressure Angle	$\alpha \; [degrees]$
Pinion Helix Angle	$\beta = \beta_d + \frac{\beta_m}{60} + \frac{\beta_s}{3600}$ $\beta \; [degrees]$ $\beta_d \; [degrees]$ $\beta_m \; [arcmin]$ $\beta_s \; [arcsec]$
Machining Force	$F \; [N]$
Motor Inertia	$J_M \; [kg \cdot cm^2]$
Motor Max. Speed During Cycle	$n_1 \; [rpm]$
Gearbox Inertia	$J_g \; [kg \cdot cm^2]$
Positioning Accuracy	$A_P \; [mm]$

Deceleration Force

$$F_{dec} = m \cdot a \cdot \eta \quad [N]$$

Force due to Gravity

$$F_w = \frac{m \cdot g \cdot \sin\left(\frac{\gamma \cdot \pi}{180}\right)}{\eta} \quad [N]$$

Max. Tangential Force on Pinion

$$F_{t\,max} = max\begin{pmatrix} F_{acc} + F_f + F_w + F, \\ F_{dec} - F_f - F_w \end{pmatrix} \quad [N]$$

Max. Radial Force on Pinion (Straight Pinion)

$$F_{r\,max} = F_{t\,max} \cdot \tan\left(\frac{\alpha \cdot \pi}{180}\right) \quad [N]$$

Max. Radial Force on Pinion (Helical Pinion)

$$F_{r\,max} = \frac{F_{t\,max} \cdot \tan\left(\frac{\alpha \cdot \pi}{180}\right)}{\cos\left(\frac{\beta \cdot \pi}{180}\right)} \quad [N]$$

Max. Axial Force on Pinion

$$F_{ax\,max} = F_{t\,max} \cdot \tan\left(\frac{\beta \cdot \pi}{180}\right) \quad [N]$$

Total Inertia as Seen by the Motor

$$J_{T1} = 91.2 \cdot m \cdot \left(\frac{v}{n_1}\right)^2 + \frac{J_g}{10000} \quad [kg \cdot m^2]$$

Load to Motor Inertia Ratio

$$\Lambda = \frac{10000 \cdot J_{T1}}{J_M + J_g + J_P}$$

Traversed Distance

$$L = v \cdot \left(\frac{1}{2} \cdot t_a + t_{cs} + \frac{1}{2} \cdot t_d\right)$$

Linear Acceleration

$$a=\frac{v}{t_a} \quad \left[\frac{m}{s^2}\right]$$

Linear Deceleration

$$d=\frac{v}{t_d} \quad \left[\frac{m}{s^2}\right]$$

Pinion Rotational Speed

$$n_2=\frac{60\cdot v}{\pi\cdot\frac{d}{1000}} \quad [rpm]$$

Maximum Required Motor Power

$$P_{1Max}=\frac{T_{2max}\cdot n_2}{9550} \quad [kW]$$

Maximum Required Motor Torque

$$T_{1Max}=\frac{P_{1Max}\cdot 9550}{n_1} \quad [N\cdot m]$$

Ideal Gearbox Ratio

$$i=\frac{n_1}{n_2}$$

Ideal Gearbox Backlash

$$\Delta\phi=\frac{60\cdot A_p}{\frac{2\cdot\pi}{360}\cdot\frac{d}{2}} \quad [arcmin]$$

Note: This considers only the internal backlash of the gearbox, not the backlash introduced by the rack and pinion system

Required Torque on Pinion

$$T_{2max}=\frac{d\cdot F_{tmax}}{2000} \quad [N\cdot m]$$

Required Torque on Pinion, Adjusted for Service Factor

$$T_{2KA}=T_{2max}\cdot K_A \quad [N\cdot m]$$